Trajectory Tracking Control of Robot Based On High-Gain Observer

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Abstract: In order to improve the tracking control accuracy and robustness an attitude controller for robot was designed, a nonlinear high-gain observer was designed to estimate the linear velocity of the robot, and a control method based on the finite time control of Output feedback control was proposed to realize the high precision tracking control. Based on Lyapunov theory, the stability of the closed-loop system was proved. Simulation results show that a rapid trajectory tracking performance is guaranteed without linear velocity feedback, the engineering effectiveness and feasibility of the method were illustrated by experiments.

Keywords: robot; high-gain observer; feedback control; tracking control

1. Introduction

In recent years, with the continuous innovation and breakthrough in the field of robot technology, not only has it created huge economic wealth for the society, it has also promoted the process of high industrial automation, and it has also improved people's living standards. With the rapid development of the industrial robot industry, people have higher and higher requirements for high-speed and high-precision control of robots, therefore, the research of robot advanced control algorithm has become a research hotspot in the field of automation.

In the literature [1], an attitude controller for quadrotor UAV was designed based on terminal sliding mode control method to ensure a rapid orientation tracking, where a nonlinear function was introduced to design a terminal sliding mode surface, so that the tracking error could converge to zero in finite time. In the literature [2], an induction motor flux observer based on high gain observation technology is proposed, which can achieve any precision error. The stability analysis of the nonlinear normal high gain observer is carried out. The literature [3-4] has designed time-varying high gain observers to estimate boundary perturbations and offset it by state feedback to stabilize the system. In the literature [5], only when the position sensor, current and speed and its derivative signals are obtained through the state observer, the position tracking control of the DC motor is realized, making the control system has good control performance and is easy to achieve with hardware. The literature [6-7] uses a high-gain observer to state estimate the velocity signal, based on the direct measurement of the ocean platform position signal and the velocity signal indirectly obtained from the state estimation. The literature [8-9] designed high-gain observers to measure the angular velocity and displacement speed, such as the robot, to estimate its state. Compared with the traditional control methods, the robot has better dynamic quality in self-balance and track tracking, and has good application prospects. In the literature [10], sliding mode variable structure control is used to ensure the establishment of the manifold for the fast subsystem. In order to avoid direct measurement of curvature change, introduce high gain observer to estimate it. Lyapunov stability principle proves overall stability, and give upper bound on small parameter.

In order to further improve robot control accuracy and anti-interference performance, considering nonlinear high gain observer, an output feedback control method based on finite time stability is proposed to realize high-precision tracking control of the robot.

2. Robot system design

2.1. Robot Structure Design

The robot system is composed of the robot body, robot arm and controller. Its body structure belongs to the spatial articulated open motion chain. The controller is controlled by the program code executed by the upper position computer for the trajectory movement of the robot joint. The joint drive of the robot is completed by the servo motor, therefore, the motion control of the robot is actually to carry out high precision control of the servo motor and then realize the high precision control of the robot joint.

The robot body structure (Fig. 1) is mainly composed of machine seat, arm, reducer, servo motor and drive. The control object of the experiment is the robot joint with plane two degrees of freedom. It reflects the precision control of the robot by the control algorithm by detecting the position of the robot joint.

Figure 1. Structure of the robot experimental platform

2.2. Design of the Robot Control System

The control system of the robot is composed of the upper computer and the controller, through the LabVIEW graphical human-computer operation interface, set the movement trajectory of the robot, and through the control algorithm to send the signal to the controller is designed to realize the trajectory control of the robot. The controller adopts the NI myRIO controller, the latest embedded system development platform launched by NI, to collect the servo motor encoder A phase and B phase pulse signal into the analog voltage signal driving the servo motor to achieve the purpose of closed-loop control.

3 High-Gain Observer Design

3.1. Finite Time Controller

The relevant theorems and derivations of the finite time controller are as follows.

Definition 1: Convergence and stability in finite time:

For the system

$$\dot{x} = f(x,t), \quad f(0,t) = 0, \quad x \in R^n \tag{1}$$

Where $f: U_o \times R \to R^n$ is continuous on $U_o \times R$, and U_o is an open area of the origin $x = 0$. Initial state $x(t_0) = x_0 \in U$ has a time dependent $T \ge 0$ on x_0 at any initial moment, so that the solution of system (1) is $x(t) = \varphi(t; t_0; x_0)$, and

$$\lim_{t \to T(x_0)} \varphi(t; t_0; x_0) = 0 \tag{2}$$

When $t \in [t_0, T(x_0)]$, $\varphi(t; t_0, x_0) \in U\{0\}$, the equilibrium point of system (1) converges in finite time. Therefore, when $t > T(x_0)$, $\varphi(t; t_0, x_0) = 0$.

If $U = R^n$, then the equilibrium point of system (1) is globally stable in finite time.

Lemma 1 [11] considers the following system

$$\dot{x} = f(x) + \hat{f}(x), \quad f(0) = 0, x \in R^n \tag{3}$$

Where, $f(x)$ is a continuous homogeneous vector field with homogeneous degrees of freedom k<0 with respect to (r1,…, rn) (ri>0, i = 1,…, n), and $\hat{f}(x)$ satisfies $\hat{f}(0) = 0$. Suppose $x = 0$ is asymptotically stable equilibrium point of the system $\dot{x} = f(x)$, then $x = 0$ is the equilibrium point where the system is locally stable within a finite time. If

$$\lim_{\varepsilon \to 0} \frac{\hat{f}_i(\varepsilon^{r_i} x_1, \cdots, \varepsilon^{r_i} x_n)}{\varepsilon^{k + r_i}} = 0, \quad i = 1, \cdots, n, \forall x \ne 0. \tag{4}$$

Lemma 2[12] A closed-loop system that is globally asymptotically stable and locally stable in finite time is globally stable in finite time.

Definition 3 If the vector $x = [x_1, x_2, \cdots, x_n]^T$, then define

$$|x|^\alpha = [|x_1|^\alpha, \cdots, |x_n|^\alpha]^T, \ \mathrm{sig}^\alpha(x) = [\mathrm{sgn}(x_1)|x_1|^\alpha, \cdots, \mathrm{sgn}(x_n)|x_n|^\alpha]^T \tag{5}$$

Where α> 0.

Using the separation principle, first design a state feedback controller to make the system meet the control requirements, and then design a high-gain observer to replace x with the estimated $\hat{x}$, thereby obtaining the output feedback controller.

Define $e = q - q_d$, $\dot{e} = \dot{q} - \dot{q}_d$, and let $x_1 = e$, $x_2 = \hat{x}_1 = \dot{e}$, then the dynamics equation of the robot can be rewritten as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = M^{-1}(q)\left[\tau - C(q,\dot{q})(\dot{q}) - G(q)\right] - \ddot{q}_d \end{cases} \tag{6}$$

Then the design controller is

$$\tau = -K_p Sig^{\alpha_1}(x_1) - K_d Sig^{\alpha_2}(x_2) + M(q)\ddot{q}_d + C(q,\dot{q}_d)\dot{q}_d + G(q) \tag{7}$$

Where, K_p and K_d are positive definite diagonal constant matrices, 0 < α_1<1, α_2=2α_1/(α_1+1).

Demonstration: Substitute formula (7) into formula (6) to get

$$M(q)\ddot{e} + \left[C(q,\dot{q}) + C(q,\dot{q}_d)\right]e + K_p Sig^{\alpha_1}(e) + K_d Sig^{\alpha_2}(\dot{e}) = 0 \tag{8}$$

Or it is written as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1 + q_d)[(C(x_1 + q_d, x_2) + C(x_1 + q_d, \dot{q}_d))x_2 \\ \quad + K_p Sig^{\alpha_1}(x_1) + K_d Sig^{\alpha_2}(x_2)] \end{cases} \tag{9}$$

First, let's prove the semi-global asymptotic stability:

Consider the following Lyapunov function

$$V = \frac{1}{2}\dot{e}^T M(q)\dot{e} + \int_0^e K_p Sig^{\alpha_1}(\rho)d\rho \tag{10}$$

By derivation, there is:

$$V = \frac{1}{2}\dot{e}^T \dot{M}(q)\dot{e} + \dot{e}^T M(q)\ddot{e} + \dot{e}^T K_p Sig^{\alpha_1}(e) \tag{11}$$

According to formula (8) and characteristics of the robot, there is

$$V = -\dot{e}^T K_d Sig^{\alpha_2}(\dot{e}) - \dot{e}^T C(q,\dot{q}_d)\dot{e} \tag{12}$$

Since

$$\begin{aligned} &\dot{e}^T C(q,\dot{q}_d)\dot{e} \le c_2 V_M \|\dot{e}\|^2 \\ &\dot{e}^T K_d Sig^{\alpha_2}(\dot{e}) \ge K_{dm}\dot{e}^T \dot{e}, \ \forall \|\dot{e}\| \le 1 \end{aligned} \tag{13}$$

There is

$$V \le -\theta \dot{e}^T K_d Sig^{\alpha_2}(\dot{e}) - \left[(1-\theta)K_{dm} - c_2 V_M\right]\|e\|^2, \ \ \forall \|\dot{e}\| \le 1 \tag{14}$$

Where, $0 < \theta < 1$. If we choose $K_{dm} > c_2 V_M / (1-\theta)$, then there is $\dot{V} \le 0, \forall \|\dot{e}\| \le 1$. Because $\dot{V} \equiv 0$, it means that $\dot{e} \equiv 0$. According to LaSalle's invariant set theory, when $t \to \infty$, there are $e \to 0, \dot{e} \to 0$ in the neighborhood of the equilibrium point, and the attraction domain can be made arbitrarily small by increasing K_d. Therefore, the closed-loop system is semi-global stable in finite time.

Then, let's prove the local finite time stability of the system:

From equation (9), we know that x=0 is the equilibrium point of the system, but system (9) is inhomogeneous. Therefore, we can rewrite this equation as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1+q_d)\left[K_p Sig^{\alpha_1}(x_1)+K_d Sig^{\alpha_2}(x_2)\right]+\hat{f}(x) \end{cases} \quad (15)$$

Where

$$\hat{f}(x) = -M^{-1}(x_1+q_d)(C(x_1+q_d,x_2)+C(x_1+q_d,\dot{q}_d))x_2 \\ -\tilde{M}(x_1,q_d)\left[K_p Sig^{\alpha_1}(x_1)+K_d Sig^{\alpha_2}(x_2)\right]$$

$$\tilde{M}(x_1,q_d) = M^{-1}(x_1+q_d)-M^{-1}(q_d)$$

Obviously, it is easy to prove that

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -M^{-1}(x_1+q_d)\left[K_p Sig^{\alpha_1}(x_1)+K_d Sig^{\alpha_2}(x_2)\right] \end{cases} \quad (16)$$

There is the degree of homogeneity $\kappa = \alpha_1 - 1 < 0$ with respect to $(r_{11}, r_{12}, \cdots, r_{1n}, r_{21}, r_{22}, \cdots, r_{2n})$. Where, r_{1i}=r_1=2, r_{2i}= r_2= α_1+1. Since $M^{-1}(x_1+q_d)$ and $C(x_1+q_d, x_2)$ are smooth, and $\kappa < 0$, there is

$$\lim_{\sigma\to 0}\frac{-M^{-1}(\sigma^{r_1}x_1+q_d)\left[(C(\sigma^{r_1}x_1+q_d,\sigma^{r_2}x_2)+C(\sigma^{r_1}x_1+q_d,\dot{q}_d))\sigma^{r_2}x_2\right]}{\sigma^{\kappa+r_2}} \\ = -M^{-1}(q_d)\left(C(q_d,0)+C(q_d,\dot{q}_d)\right)x_2\lim_{\sigma\to 0}\sigma^{-\kappa} \\ = 0 \quad (17)$$

For $\tilde{M}(x_1,q_d)$,we can know from the mean value theorem that

$$\tilde{M}(\sigma^{r_1}x_1,q_d) = M^{-1}(\varepsilon^{r_1}x_1+q_d)-M^{-1}(q_d) = O(\sigma^{r_1}) \quad (18)$$

Hence, there is

$$\lim_{\sigma\to 0}\frac{-\tilde{M}(\sigma^{r_1}x_1+q_d)\left[K_p Sig^{\alpha_1}(\sigma^{r_1}x_1)+K_d Sig^{\alpha_2}(\sigma^{r_2}x_2)\right]}{\sigma^{\kappa+r_2}} \\ = \lim_{\sigma\to 0}O(\sigma^{r_1-\kappa-r_2})\left[K_p Sig^{\alpha_1}(\sigma^{r_1}x_1)+K_d Sig^{\alpha_2}(\sigma^{r_2}x_2)\right] \\ = 0 \quad (19)$$

Since $-\kappa = 1-\alpha_1 > 0$, $r_1-\kappa-r_2 = 2(1-\alpha_1) > 0$, for a given $x = (x_1^T, x_2^T)^T \in R^{2n}$, there is

$$\lim_{\sigma\to 0}\frac{\hat{f}\left(\sigma^{r_1}x_1,\sigma^{r_2}x_2\right)}{\sigma^{\kappa+r_2}} = 0 \quad (20)$$

Therefore, according to Lemma 1 and 2, the closed-loop system is locally stable in a finite time, and the demonstration is completed.

3.2. High Gain Observer

In the actual robot control system, it is necessary to measure the speed signal of the robot joints, but it is difficult to access the speed information or the measurement cost is quite high. To this end, a nonlinear high-gain observer is used to estimate the linear speed information of the robot online. A stable output feedback control method based on finite time is proposed to achieve high-precision tracking control of the robot.

Let's define $p_1 = q,\ p_2 = \dot{q}$. Where p_1 is measurable, and p_2 is not measurable, which will be estimated by the high-gain observer, so there is

$$\begin{cases} \dot{p}_1 = p_2 \\ \dot{p}_2 = H(\tau, p_1, p_2, \dot{p}_2) \end{cases} \quad (21)$$

Where

$$H(\tau, p_1, p_2, \dot{p}_2) = M(p_1)^{-1}\left[\tau - C(p_1,p_2)p_2 - G(p_1)\right]$$

Suppose that $\hat{p}_1, \hat{p}_2$ are the estimated values of p_1, p_2 respectively, then the high-gain observer of the dual-joint robot system is:

$$\begin{cases} \dot{\hat{p}}_1 = \hat{p}_2 + \dfrac{h_1}{\varepsilon}(p_1-\hat{p}_1) \\ p_2 = H + \dfrac{h_2}{\varepsilon^2}(p-\hat{p}_1) \end{cases} \quad (22)$$

Where, h_1, h_2 meet the Hurwitz condition, and ε is a small normal number, so the tracking control method based on the high-gain observer can be obtained:

$$\tau = -K_p Sig^{\alpha_1}(e_1) - K_d Sig^{\alpha_2}(\hat{e}_2) + M(q)\ddot{q}_d + C(q,q_d)q_d + G(q) \quad (23)$$

Where, $\hat{e}_2 = \dot{\hat{q}} - \dot{q}_d$.

Theorem 1 for a robot system (6) with uncertainty, if the observer is designed as equation (22) and the controller is designed as equation (23), when $K_{dm} > c_2 V_M / (1-\theta)$ is selected, the closed-loop system is locally stable in finite time.

4. Experimental Results and Analysis

To illustrate the engineering effectiveness and feasibility of the algorithm designed (abbreviated as "OFTC"), the performance of the design controller was tested experimentally. The dynamic equation of the second-joint mechanical arm is

$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -2b\dot{q}_2 & -b\dot{q}_2 \\ b\dot{q}_1 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (24)$$

Where

$$m_{11} = p_1 + p_2 + 2p_3\cos q_2 - 2p_4\sin q_2$$
$$m_{12} = p_2 + p_3\cos q_2 - p_4\sin q_2$$
$$m_{22} = p_2$$
$$b = p_3\sin q_2 + p_4\cos q_2$$
$$f_1 = f_{v1}\dot{q}_1 + f_{e1}\,\mathrm{sgn}(\dot{q}_1)$$
$$f_2 = f_{v2}\dot{q}_2 + f_{e2}\,\mathrm{sgn}(\dot{q}_2)$$

p_1, p_2, p_3, p_4 is the minimum inertial parameter of the robot, f_{v1} , f_{e1} viscous friction and coulomb friction coefficient of joint 1, and also f_{v2} , f_{e2} the coefficient of friction of joint 2.

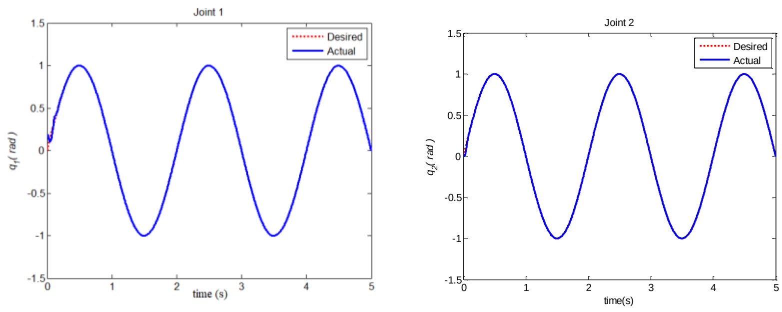
The actual parameters of the robot are p_1=0.0289kgm^2, p_2=0.0029kgm^2, p_3=-0.0035kgm^2, p_4=0.0079kgm^2, f_{v1} =0.5279, f_{e1} =0.6996, f_{v2} =0.001, f_{e2} =0.002. Its estimate is p_1=0.0268kgm^2, p_2=0.01528kgm^2, p_3=–0.00025kgm^2, p_4=0.0041kgm^2, f_{v1} =0.6144, f_{e1} =0.8575, f_{v2} =0.0120, f_{e2} =0.0447. The given reference track signal is q_{d1}=sin (2πt), q_{d2}=sin(2πt). The initial state of the system is q_1 (0) = 0.2rad, q_2 (0) = 0.2rad. External interference with the $\tau_d = [0.2 \sin(10t), 0.1 \sin(10t)]^T$.

It is compared with the finite time tracking controller proposed in reference [11], the finite time controller (FIDC) is:

$$\tau = M_0(q)\left[\ddot{q}_d - K_p Sig(e)^{\alpha_1} - K_d Sig(\dot{e})^{\alpha_2}\right] + C_0(q,\dot{q})\dot{q} + F_0(\dot{q}) \quad (25)$$

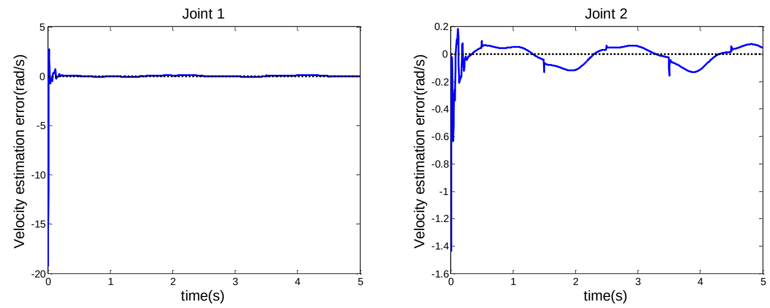
Where K_p = diag (1000, 1000), K_d = diag(500, 500), α_1 = 0.7, $\alpha_2 = 2\alpha_1/(\alpha_1+1) = 0.8235$, $F_0 = [f_1, f_2]^T$.

The parameters of the control algorithm (OFTC) were set to K_p = diag(100, 100), K_d = diag(50, 50), α_1 = 0.8, and the observer parameter was set to $h_1 = 1$, $h_2 = 1$, $\varepsilon = 0.01$.



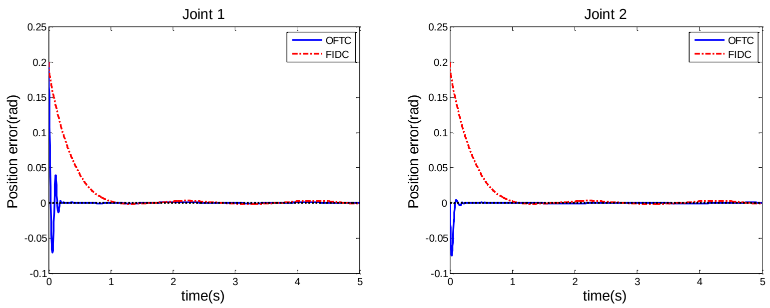
(a) (b)

Figure 2. Trajectory tracking



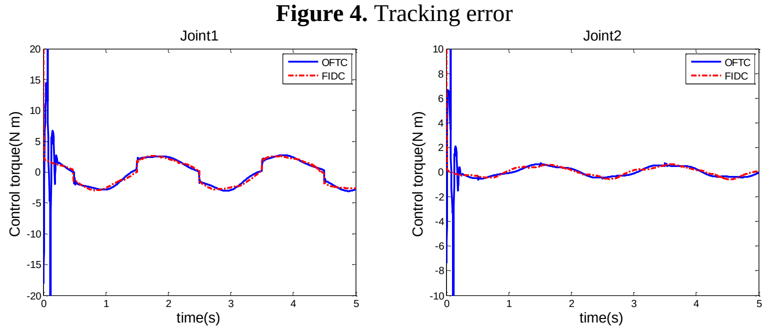
(a) (b)

Figure 3. Speed estimation error



(a) (b)

Figure 4. Tracking error



(a) (b)

Figure 5. Input torque

Figure 2 and Figure 4 are the trajectory tracking and tracking error of the two joints of the robot, respectively, and Figure 3 shows the velocity estimation error of the robot joint. Analysis of Figure 4 shows that the proposed nonlinear high-gain observer has good transient response properties and small stability error, which can estimate joint velocity signals accurately in real time online. Figure 5 shows the control input moments of the two joints of the robot system, and the comparison shows that the OFTC algorithm is better robust to kinetic uncertainties and external interference.

5. Conclusion

In order to further improve the high-speed tracking control of a given attitude, we propose a finite time stable track tracking method, design the joint speed information control algorithm, solve the output feedback problem in the robot system, and still obtain satisfactory control performance in complex nonlinear robot systems with multiple input, multiple output, strong coupling and uncertainty.

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